Aircraft Flight Controls Design Using Output Feedback

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An approach is given for designing compensators of specified structure for shaping the closed-loop step response that uses linear quadratic output-feedback techniques. This approach results in controllers that take advantage of the wealth of experience in aircraft controls design. The correct initial conditions for determining the output-feedback gains are not uniformly distributed as is traditionally assumed, but are shown to be explicitly given in terms of the step command magnitude. Arbitrary systems are treated, not only those with integrators in the forward paths, by adding a term to the performance index that weights the steady-state error. Necessary conditions are derived that may be used in a gradient-based routine to determine the optimal control gains.

I. Introduction

IN aircraft control, an important problem is that of obtaining desirable closed-loop performance specifications using a compensator of desired structure. There are often sound engineering considerations that dictate the required form of the compensator. For instance, a washout circuit may be required, or integrators may be required in some of the forward paths for zero steady-state error.

The modern linear quadratic (LQ) control theory is powerful and can guarantee closed-loop stability, but if it is naively applied the resulting compensator can have a high order and no structure. It is well-known (see, e.g., Ref. 1) that if the compensator is required to have a specified structure, the selection of the control gains is an *output-feedback* problem, not a full-state-feedback problem.

In the output-feedback problem, the optimal control gains depend on the initial state. It is traditional to assume that the initial conditions $x(0)$ are uniformly distributed so that $E\{x(0)x^T(0)\}$ is known, where $E\{\ \}$ is the expected value.[2-5]

Aircraft closed-loop performance specifications are often given in terms of time-domain criteria.[6] Since these criteria are closely tied to the step response, it is reasonable to select the control gains to yield a suitable closed-loop step response. This is the servo design problem.

In this paper, an approach is given for designing servo compensators of a desirable structure that is based on LQ output-feedback techniques. First, the technique for designing a compensator of desired structure is reviewed, which results in an output-feedback step-response shaping problem. A controller structure is obtained that is sensible in terms of the wealth of experience in aircraft controls design. It does not suffer from the usual problems of noncausality[7,8] of the optimal LQ tracker, nor does it require the addition of an extra term to ensure zero steady-state error.[7] It allows the position-

ing of *both the poles and zeros* to obtain suitable closed-loop response.

It is shown that the correct initial conditions for the step-response problem are not uniformly distributed, but have a specific value that is easily computed.

Attention is not restricted to type I systems, but one is able to obtain suitable step responses for arbitrary systems by adding a term to the performance index (PI) that weights the steady-state error. It is well-known from classical control that acceptable step responses may often be obtained with simplified controllers that do not use integrators in all the feedforward paths.[9]

To avoid the usual observability assumptions in the LQ regulator,[8] a weighting term of the form t^k is used in the PI. This goes along with the intuition of classical control, which used time weighting in the ITAE and ISTSE PIs to suppress the effects of a slow pole by moving it to the left in the s plane.[9]

Necessary conditions for optimality are obtained; these may be used in a gradient-based minimization algorithm for determining the optimal output-feedback gains.

The robustness of the design to low-frequency disturbances and high-frequency unmodeled dynamics[10] may easily be verified using plots of the loop gain singular values.[11]

To demonstrate the approach, a glide slope coupler design example is offered.

The proposed technique is in keeping with the philosophy in Refs. 1 and 12. However, a different problem formulation and a different PI are used.

II. Incorporating a Servo of Desired Structure

In aircraft controls design, there is a wealth of experience and knowledge that dictates in many situations what sort of compensator dynamics would yield good performance from the point of view of both the controls engineer and the pilot. For example, a washout circuit may be required, or it may be necessary to augment some feedforward channels with integrators to obtain a steady-state error of exactly zero.

A dynamic compensator of prescribed structure may be incorporated into the system description as follows.[1] See Fig. 1.

Let the plant be described by

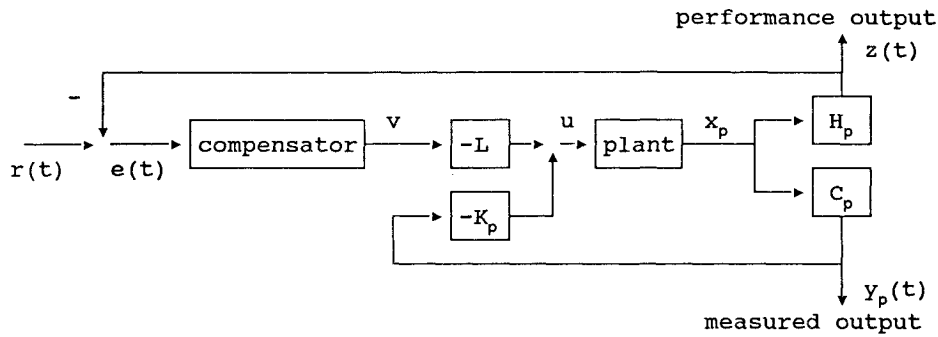$$\dot{x}_p = A_p x_p + B_p u_p \tag{1}$$

$$y_p = C_p x_p \tag{2}$$

Fig. 1   Plant with compensator of desired structure.

with state $x_p(t)$, control input $u_p(t)$, and $y_p(t)$ the *measured plant output*. In addition, let

$$z = H_p x_p \tag{3}$$

be a *performance output*, which must track the given *reference input* $r(t)$. The performance output $z(t)$ is not generally equal to $y_p(t)$.

The dynamic compensator has the form

$$\dot{w} = F_c w + Ge \tag{4a}$$

$$v = Dw + Je \tag{4b}$$

with state $w(t)$, output $v(t)$, and input equal to the *tracking error*

$$e(t) = r(t) - z(t) \tag{5}$$

F_c, G, D, and J are matrices chosen to include the desired structure in the compensator.

The allowed form for the plant control input is

$$u = -K_p y_p - Lv \tag{6}$$

where the constant gain matrices K_p and L are to be chosen in the controls design step to result in satisfactory tracking of $r(t)$. This formulation allows for both *feedback and feedforward compensator dynamics*. Note that any dynamics in the feedforward path are termed feedforward, even though they may be inside the control loops, since they are able to move the system zeros.

These dynamics and output equations may be written in augmented form as

$$\frac{d}{dt}\begin{bmatrix} x_p \\ w \end{bmatrix} = \begin{bmatrix} A_p & 0 \\ -GH_p & F_c \end{bmatrix} \begin{bmatrix} x_p \\ w \end{bmatrix} \begin{bmatrix} B_p \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ G \end{bmatrix} r \tag{7}$$

$$\begin{bmatrix} y_p \\ v \end{bmatrix} = \begin{bmatrix} C_p & 0 \\ -JH_p & D \end{bmatrix} \begin{bmatrix} x_p \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ J \end{bmatrix} r \tag{8}$$

$$z_p = [H_p \quad 0] \begin{bmatrix} x_p \\ w \end{bmatrix} \tag{9}$$

and the control input may be expressed as

$$u = -[K_p \quad L] \begin{bmatrix} y_p \\ v \end{bmatrix} \tag{10}$$

Note that, in terms of the augmented plant/compensator state description, the admissable controls are represented as a *constant output feedback* $[K_p \quad L]$. In the augmented description, all matrices are known except the gains K_p and L, which need to be selected to yield an acceptable closed-loop performance.

A comment on the compensator matrices F_c, G, D, and J is in order. Often, these matrices are completely specified by the structure of the compensator. Such is the case, for instance, if the compensator contains integrators. However, if it is desired to include a washout or a lead-lag, it may not be clear exactly how to select the time constants. In such cases, engineering judgment will usually give some insight. However, it may sometimes be necessary to go through the proposed design, and then if required return to readjust F_c, G, D, and J and perform the design again. As will be seen in the design example, however, the zeros of a lead-lag are not a problem, for they depend on the feedforward gain L, and so are automatically adjusted by the proposed algorithm.

III.   LQ Output-Feedback Formulation

By defining the composite state $x = [x_p^T \ w^T]^T$, the composite measured output $y = [y_p^T \ v^T]^T$, and appropriate matrix variables to streamline the notation, it is seen that the augmented Eqs. (7–9) that contain both the plant and the compensator are of the form

$$\dot{x} = Ax + Bu + Er \tag{11}$$

$$y = Cx + Fr \tag{12}$$

$$z = Hx \tag{13}$$

with $y(t)$ the measured output and $z(t)$ the performance output which is required to track the reference input $r(t)$.

In this description, take the state $x(t) \in R^n$, control input $u(t) \in R^m$, reference input $r(t) \in R^q$, $y(t) \in R^p$, and $z(t) \in R^q$. The admissible controls, Eq. (10), are proportional output feedbacks of the form

$$u = -Ky = -KCx - KFr \tag{14}$$

with constant gain K to be determined.

Using these equations, the closed-loop system is found to be

$$\dot{x} = (A - BKC)x + (E - BKF)r \tag{15}$$

This formulation differs sharply from the traditional formulations of the optimal tracker problem.[7,8] Note that Eq. (14) includes both feedback and *feedforward* terms, so that both the closed-loop poles *and zeros* may be affected by varying the gain K. Note also that, in contrast to Ref. 7, the allowed form of Eq. (6) for the control has been specified at the outset (cf. Ref. 1).

Since the performance specifications of aircraft are usually given in terms of time-domain criteria,[6] and these criteria are closely related to the step response, it shall be assumed henceforth for design purposes that the reference input $r(t)$ is a step command with magnitude r_0. It should be clearly realized, however, that although our selection of K is based on step-response shaping, the tracker will work for *any* reference command.

Denote steady-state values by overbars and deviations from the steady-state values by tildes. Then, the state, output, and control deviations are given by

$$\tilde{x}(t) = x(t) - \bar{x} \tag{16}$$

$$\tilde{y}(t) = y(t) - \bar{y} = K\tilde{x} \tag{17}$$

$$\tilde{z}(t) = z(t) - \bar{z} = H\tilde{x} \tag{18}$$

$$\tilde{u}(t) = u(t) - \bar{u} = -KCx - KFr_0$$
$$-(-KC\bar{x} - KFr_0) = -KC\tilde{x}(t) \tag{19a}$$

or

$$\tilde{u} = -K\tilde{y} \tag{19b}$$

The tracking error $e(t) = r(t) - z(t)$ is given by

$$e(t) = \tilde{e}(t) + \bar{e} \tag{20}$$

with

$$\tilde{e}(t) = e(t) - \bar{e} = (r_0 - Hx) - (r_0 - H\bar{x}) = -H\tilde{x}$$

or

$$\tilde{e} = -\tilde{z} \tag{21}$$

Since in any acceptable design the closed-loop plant will be asymptotically stable, A_c is nonsingular. According to Eq. (15), at steady state

$$0 = A_c\bar{x} + B_c r_0 \tag{22}$$

so that the steady-state response $\bar{x}$ is

$$\bar{x} = -A_c^{-1}B_c r_0 \tag{23}$$

and

$$\bar{e} = r_0 - H\bar{x} = (I + HA_c^{-1}B_c)r_0 \tag{24}$$

Using Eqs. (16), (19), and (23) in Eq. (15) the dynamics of the state deviation are seen to be

$$\dot{\tilde{x}} = A_c\tilde{x} \tag{25}$$

$$\tilde{y} = C\tilde{x} \tag{26}$$

$$\tilde{z} = H\tilde{x} = -\tilde{e} \tag{27}$$

and the control input to the *deviation system*, Eq. (25), is

$$\tilde{u} = K\tilde{y} \tag{28}$$

Thus, the step-response shaping problem has been converted to a regulator problem for the deviation system.

Again, note the difference between the proposed approach and traditional ones (e.g., Ref. 7). Once the gain K in Eq. (28) has been found, the control for the plant is given by Eq. (14), which inherently has both feedback and feedforward terms. Thus, no extra feedforward term needs to be added to make $\bar{e}$ zero.

To make the tracking error $e(t)$ in Eq. (20) small, it is proposed to attack two equivalent problems: the problem of regulating the error deviation $\tilde{e}(t) = -\tilde{z}(t)$ to zero, and the problem of making the steady-state error $\bar{e}$ small.

Note that a type I system is not assumed, so that $\bar{e}$ may be nonzero. This can be important in aircraft controls, where it may not be desirable to force the system to be of type I by augmenting all control channels with integrators. This augmentation complicates the servo structure. Moreover, it is well-known from classical control theory that suitable step responses may often be obtained without resorting to inserting integrators in all of the feedforward channels. An additional consideration is that the problems involving integrator windup in implementing type I servos are well documented.[13]

To make both the error deviation $\tilde{e}(t) = -H\tilde{x}(t)$ and the steady-state error $\bar{e}$ small, one may select K to minimize the PI:

$$J = \frac{1}{2}\int_0^\infty (t^k\tilde{e}^T\tilde{e} + \tilde{u}^TR\tilde{u})\ dt + \frac{1}{2}\bar{e}^TV\bar{e} + \frac{1}{2}\sum_i\sum_j g_{ij}k_{ij}^2 \tag{29}$$

with $R > 0$, $V \geq 0$, and $g_{ij} \geq 0$ as the design parameters. The integrand is the standard quadratic PI with, however, time weighting t^k of the error deviation; V is a weighting on the steady-state error, and g_{ij} is a weight on element k_{ij} of the gain matrix K. Note that the PI integrand weights the error and control *deviations*, and not the errors and controls themselves.

The time-varying weighting t^k in the PI places a heavy penalty on errors that occur late in the response, and is thus very effective in suppressing the effect of a slow pole, as well as in eliminating lightly damped settling behavior. This is well known from classical control theory, where the idea was used in ITAE and ISTSE PIs.[9] Time weighting has been used for the regulator problem in Refs. 14-16.

It is known that, if time-varying weighting is used in the PI, the optimal feedback gains are time-varying.[17] However, here it is assumed that K is time-invariant, so that a suboptimal solution is obtained which is usually more suitable for aircraft controls purposes. (Note that it is extremely difficult to gain-schedule time-varying gains.) This has been the common approach in the literature.[14-16]

To make $\bar{e}$ smaller, V may be selected larger. The gain weights g_{ij} may be selected large to zero certain elements of the matrix K in order to obtain more structure in the controller.[4] If the system is of type I, which may be ensured by adding integrators in the feedforward paths, then V may be set to zero since $\bar{e}$ is zero. However, even if this is not the case the proposed approach will allow selection of K to make $\bar{e}$ *small enough* (see the design example).

If the system is of type 0, then it is known from classical control theory that to make the steady-state error $\bar{e}$ exactly zero it is generally necessary to increase the feedback gains to infinity. Thus, generally, as V is increased the elements of K will increase without bound. This effect may be counteracted by selecting larger values of g_{ij}. What this amounts to is a design technique that allows tradeoffs between small steady-state errors and reasonable values of the gain K by selecting V and g_{ij}.

Making the error deviation $\tilde{e}(t)$ small improves the transient response, while making the steady-state error $\bar{e}(t)$ small improves the steady-state response. These effects also involve a tradeoff, so that there is also a design tradeoff between selecting large values of k and large values of V.

It is often adequate to select $R = rI$ and $V = vI$, with r and v being scalars. This simplifies the design since now only a few parameters must be tuned during the interactive design process. If different control-loop bandwidths are required, then one could select $R = \text{diag}\{r_i\}$. If multiple control surface blending is required, then some of the off-diagonal terms in R will be nonzero.

It should be pointed out that the proposed approach is suboptimal in the sense that minimizing the PI does not necessarily minimize a quadratic function of the total error $e(t) = \bar{e} + \tilde{e}(t)$.

Using Eqs. (27) and (28), the PI may be written as

$$J = \frac{1}{2} \int_0^\infty (t^k \tilde{x}^T Q \tilde{x} + \tilde{x}^T C^T K^T R K C \tilde{x}) \, dt$$

$$+ \frac{1}{2} \bar{e}^T V \bar{e} + \frac{1}{2} \sum_i \sum_j g_{ij} k_{ij}^2 \tag{30}$$

with $Q = H^T H$.

An important advantage of the time weighting t^k should be clearly understood. Namely, if $k > 1$, it is not necessary for (A,H) to be observable to obtain good designs with closed-loop stability. This will be mentioned again in the next section.

IV.   Necessary Conditions for Optimality

As is well known,[18] for any fixed value of K, the value of the PI is given by

$$J = \frac{1}{2} \tilde{x}^T(0) P_k \tilde{x}(0) + \frac{1}{2} \bar{e}^T V \bar{e} + \frac{1}{2} \sum \sum g_{ij} k_{ij}^2 \tag{31}$$

with $P_k \geq 0$ the solution to the Lyapunov equation set:

$$0 = f_0 \equiv A_c^T P_0 + P_0 A_c + Q$$

$$0 = f_1 \equiv A_c^T P_1 + P_1 A_c + P_0$$
$$\vdots$$
$$0 = f_{k-1} \equiv A_c^T P_{k-1} + P_{k-1} A_c + P_{k-2}$$

$$0 = f_k \equiv A_c^T P_k + P_k A_c + k! P_{k-1} + C^T K^T R K C \tag{32}$$

and $\bar{e}$ is given by Eq. (24).

In the case of the LQ regulator, it is traditional to assume that the initial conditions are uniformly distributed on a surface with known characteristics,[2-5] that is, that $E\{\tilde{x}(0)\tilde{x}^T(0)\}$ is known, where $E\{\ \}$ is the expected value. Then, $E\{J\}$ is minimized instead of J. This is a satisfactory assumption for the regulator problem, but it is unsatisfactory for the tracker problem. In the latter situation, the system starts at rest and must achieve a given final state that is dependent on the reference input, namely, Eq. (23). To find the correct value of $\tilde{x}(0)$, note that, since the plant starts at rest [i.e., $x(0) = 0$], according to Eq. (16),

$$\tilde{x}(0) = -\bar{x} \tag{33}$$

so that the optimal cost of Eq. (31) becomes

$$J = \frac{1}{2} \bar{x}^T P_k \bar{x} + \frac{1}{2} \bar{e}^T V \bar{e} + \frac{1}{2} \sum \sum g_{ij} k_{ij}^2$$

or

$$J = \frac{1}{2} \text{tr}(P_k X) + \frac{1}{2} \bar{e}^T V \bar{e} + \frac{1}{2} \sum \sum g_{ij} k_{ij}^2 \tag{34}$$

where tr( ) is the trace, P_k is given by Eqs. (32), $\bar{e}$ is given by Eq. (24), and

$$X \equiv \bar{x}\bar{x}^T = A_c^{-1} B_c r_0 r_0^T B_c^T A_c^{-T} \tag{35}$$

with $A_c^{-T} \equiv (A_c^{-1})^T$.

The optimal solution to the unit-step tracking problem, with Eq. (11) initially at rest, may now be determined by minimizing J in Eq. (34) over the gains K, subject to the constraints of Eq. (32) and the value for $\bar{e}$ from Eq. (24). This algebraic optimization problem can be solved by any well-known numerical method (cf. Refs. 19 and 20). A good approach for a fairly small number ($mp \leq 10$) of gain elements in K is the SIMPLEX minimization routine.[19] To evaluate the PI for each fixed value of K in the iterative solution procedure, one may solve Eqs. (32) for P_k using repeated calls to subroutine ATXPXA in Ref. 21 and then employ Eq. (34).

As an alternative solution procedure, one may use gradient-based techniques (e.g., the Davidon-Fletcher-Powell algorithm; see Ref. 19), which generally require fewer iterations than non-gradient-based approaches.[22] To find the gradient of the PI with respect to the gains, define the Hamiltonian

$$H = \frac{1}{2} \text{tr}(P_k X) + \text{tr}(f_0 S_0) + \cdots + \text{tr}(f_k S_k)$$

$$+ \frac{1}{2} \bar{e}^T V \bar{e} + \frac{1}{2} \sum \sum g_{ij} k_{ij}^2 \tag{36}$$

with S_k Lagrange multipliers (cf. Refs. 4 and 16).

Using some patience and the basic matrix calculus identities

$$\frac{\partial Y^{-1}}{\partial x} = -Y^{-1} \frac{\partial Y}{\partial x} Y^{-1} \tag{37}$$

$$\frac{\partial UV}{\partial x} = \frac{\partial U}{\partial x} V + U \frac{\partial V}{\partial x} \tag{38}$$

$$\frac{\partial y}{\partial x} = \text{tr} \left[\frac{\partial y}{\partial z} \cdot \frac{\partial z^T}{\partial x} \right] \tag{39}$$

(where matrices and vectors are denoted, respectively, by uppercase and lowercase symbols) necessary conditions for optimality are found to be Eqs. (32) and

$$0 = A_c S_k + S_k A_c^T + X$$

$$0 = A_c S_{k-1} + S_{k-1} A_c^T + k! S_k$$

$$0 = A_c S_{k-2} + S_{k-2} A_c^T + S_{k-1}$$
$$\vdots$$
$$0 = A_c S_0 + S_0 A_c^T + S_1 \tag{40}$$

and

$$0 = \frac{1}{2} \frac{\partial H}{\partial K} = R K C S_k C^T - B^T (P_0 S_0 + \cdots + P_k S_k) C^T$$

$$+ B^T A_c^{-T} (P_k + H^T V H) \bar{x} \bar{y}^T$$

$$- B^T A_c^{-T} H^T V r_0 \bar{y}^T + g^* K \tag{41}$$

with

$$\bar{y} = C \bar{x} + F r_0 \tag{42}$$

where g is the matrix with elements g_{ij}, and $g^* K$ is the Hadamard product defined as the matrix with entries $g_{ij} K_{ij}$.

To use these equations to find K by a gradient-minimization algorithm, for a fixed value of K, solve Eqs. (32) and (40) for P_i and S_i. Under these circumstances, $\partial J / \partial K = \partial H / \partial K$, which may be found using Eq. (41).

These equations should be compared to those in Refs. 4, 5, and 16. Note that the dependence of both X and $\bar{e}$ on the gain K has resulted in extra terms being added in Eq. (41).

The design procedure now proposed is an interactive one. In the PI, values of the design parameters $R = rI$, $V = vI$, and g_{ij} are selected and a numerical minimization routine is used to determine the feedback gain K. Then, the closed-loop poles are examined, as is the step response using a computer simulation. If these are not suitable, then the design parameters may be varied and the process repeated. The availability of good software is important, but given this the design procedure is straightforward and rapid.

It is emphasized that there are only a few design parameters in this approach: k, r, v, and sometimes g_{ij} [if one selects $R = rI$ and $V = vI$ in Eq. (29)]. Thus, it is not difficult or time-consuming to come up with good designs. Much of the simplicity of the approach derives from the fact that Q in Eq. (30) is equal to $H^T H$, which is known. Even if (H,A) is not observable (cf. Ref. 8), good designs usually result. This is

because [see Eqs. (32)] even if (H,A) is not observable, $([k!P_{k-1} + C^T K^T R K C], A)$ may be for some k.

An important point to note is that the robustness of the design to low-frequency disturbances and high-frequency unmodeled dynamics may be checked using the procedures in Refs. 10 and 11. It is only necessary to break the loop at $e(t)$ in the closed-loop system, find the loop gain, and examine the plots vs frequency of its minimum and maximum singular values.

V. Example: Glide Slope Coupler

The design technique will now be demonstrated in the design of a glide slope coupler for the longitudinal dynamics of a medium-sized transport aircraft.[23]

A. Glide Slope Geometry

The glide slope geometry is shown in Fig. 2, where d is the perpendicular distance from the glide path and V_T is the total forward velocity. The aircraft flight-path angle is γ. The glide slope angle may be interpreted as a *commanded or reference flight path angle* γ_r. It is generally 2.5 deg.

The control objectives in the glide slope coupler are to regulate to zero the off-glide path distance d and the deviation v_T from trim velocity. Then, the aircraft will remain on the glide path with the nominal trim velocity. To accomplish this, the two control inputs are throttle δ_t and elevator deflection δ_e. The outputs available for feedback are pitch rate q, pitch angle θ, velocity deviation v_T, and d, which is available from measurements taken from the ground.

According to Fig. 2, when $(\gamma + \gamma_r)$ is small the component of velocity perpendicular to the glide path is given by

$$\dot{d} = V_T \sin(\gamma + \gamma_r) \approx V_T(\gamma + \gamma_r) \qquad (43)$$

We shall assume that the velocity deviation v_T is small, and take V_T in Eq. (43) as the trim velocity. To follow the glide path, we require $\dot{d} = 0$, so that the flight-path angle γ should be equal to $-\gamma_r$; i.e., the aircraft should descend at an angle
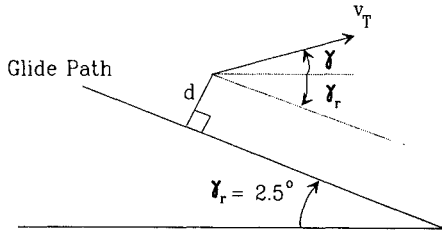

Fig. 2   Glide slope geometry.

of $-\gamma_r$. Assuming a trim velocity of $V_T = 250$ ft/s, we may use the relation $\gamma = \theta - \alpha$ to write

$$\dot{d} = V_T \theta - V_T \alpha + \frac{V_T}{57.2958} \gamma_r = V_T \theta - V_T \alpha + 4.3633 \gamma_r \quad (44)$$

with θ and α in radians and γ_r in degrees.

B. Aircraft Dynamics and Control Structure

The aircraft, actuators, and controller are shown in Fig. 3. The longitudinal dynamics of the aircraft were linearized about a trim velocity of $V_T = 250$ ft/s. Also included were a throttle actuator and an elevator actuator.

The objective is to regulate v_T and d to zero. Thus, define the performance output as

$$z = \begin{bmatrix} v_T \\ d \end{bmatrix} = Hx \qquad (45)$$

Now, examine Fig. 3, which has been drawn to show that this may be considered as a tracking problem with reference commands r_v and r_d of zero. The tracking error is $e = [e_v \ e_d]^T$ with

$$e_v = r_v - v_T \, eq \qquad (46a)$$

$$e_d = r_d - d \qquad (46b)$$

The compensators proposed are of the form

$$\frac{w_v}{e_v} = \frac{k_1}{s+5} + k_6 = k_6 \frac{[s + (5 + k_1/k_6)]}{s + 5} \qquad (47a)$$

$$u_t = -w_v \qquad (47b)$$

and

$$\frac{w_d}{e_d} = \frac{k_2}{s(s+10)} + \frac{k_3}{s} + k_7 \qquad \qquad (48a)$$

$$= k_7 \frac{[s^2 + (10 + k_3/k_7)s + (k_2 + 10k_3)/k_7]}{s(s+10)}$$

$$u_e = -w_d \qquad (48b)$$

The important point to note is that, by varying the control gains, *both the compensator gain and its zeros* are adjusted. Thus, the LQ optimization routine may adjust the zeros of the compensators, presumably inducing lead compensation where it is required.
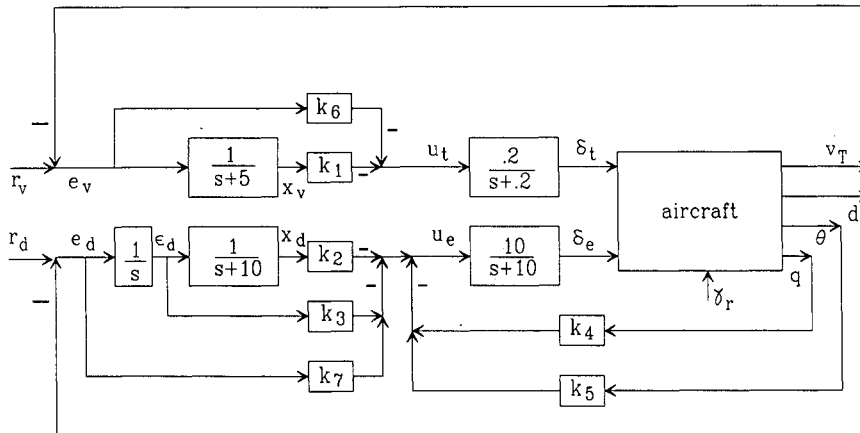

Fig. 3   Glide slope coupler.

To formulate the controller so that the gains may be determined by our output-feedback LQ approach, write the state variable representations of Eqs. (47) and (48):

$$\dot{x}_v = -5x_v + e_v = -5x_v - v_T + r_v \qquad (49)$$

$$u_t = -k_1 x_v - k_6 e_v = -k_1 x_v - k_6(-v_T + r_v) \qquad (50)$$

and

$$\dot{\epsilon}_d = e_d = -d + r_d \qquad (51)$$

$$\dot{x}_d = -10x_d + \epsilon_d \qquad (52)$$

$$u_e = -k_2 x_d - k_3 \epsilon_d - k_7 e_d = -k_2 x_d - k_3 \epsilon_d - k_7(-d + r_d) \qquad (53)$$

Now, the dynamical Eqs. (44), (49), (51), and (52) may be incorporated into the linearized aircraft system description by defining the augmented state

$$x = [v_T \ \alpha \ \theta \ q \ d \ \delta_t \ \delta_e \ x_v \ x_d \ \epsilon_d]^T \qquad (54)$$

The augmented system is described by

$$\dot{x} = Ax + Bu + Er \qquad (55)$$

with

$$A = \begin{bmatrix} -0.04 & 19.0096 & -32.1689 & 0 & 0 & 10.1 & 0 & 0 & 0 & 0 \\ -0.001 & -0.64627 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.7739 & 0 & -0.529765 & 0 & 0.02463 & -0.011 & 0 & 0 & 0 \\ 0 & -250 & 250 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -10 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -10 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (56a)$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4.3633 \\ 0.2 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad E = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad (56b)$$

The reference input is defined as

$$r = [r_v \ r_d]^T \qquad (57)$$

which is zero for the glide slope coupler, since the desired velocity deviation v_T and off-glide path distance d are equal to zero. Then, the aircraft will fly down the glide path with a total velocity equal to the trim value of $V_T = 250$ ft/s.

To incorporate the constant disturbance γ_r required in Eq. (44), the augmented input

$$u' = [u^T \ \gamma_r]^T = [u_t \ u_e \ \gamma_r]^T \qquad (58)$$

has been defined, which contains an exogenous input γ_r. For the design of the control system, only the control input $u(t)$ is used. The full input $u'(t)$ will be required only in the simulation stage, where γ_r will be set equal to 2.5 deg to obtain the desired landing approach behavior.

Equations (50) and (53) may be incorporated by defining a measured output as

$$y = [x_v \ x_d \ \epsilon_d \ q \ \theta \ e_v \ e_d]^T \qquad (59)$$

Then,

$$y = Cx + Fr \qquad (60)$$

with

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 57.2958 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 57.2958 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (61)$$

The factors of 57.2958 convert q to degrees per second and θ to degrees.

Now, according to Fig. 3, the control vector $u(t)$ is given by the output feedback

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 57.2958 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 57.2958 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$u = \begin{bmatrix} u_t \\ u_e \end{bmatrix} = -\begin{bmatrix} k_1 & 0 & 0 & 0 & 0 & k_6 & 0 \\ 0 & k_2 & k_3 & k_4 & k_5 & 0 & k_7 \end{bmatrix} y = -Ky \qquad (62)$$

which has some elements constrained to zero.

According to Eq. (45), one may write

$$z = \begin{bmatrix} v_T \\ d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x = Hx \qquad (63)$$

At this point, the glide slope coupler design problem has been cast into the LQ formulation required in this paper.

Let us motivate the control structure. To obtain zero steady-state error in $v_T(t)$ and $d(t)$, integrators could be added in each of the forward error paths. However, adding compensator poles near the origin makes the problem of stabilization

more difficult. Since it is important to keep d exactly at zero, an integrator has been added in the forward path corresponding to the tracking error in d. A small enough error in v_T can then be obtained without a forward-path integrator by using weighting of the steady-state error in the PI.

An additional consideration for including a forward-path integrator in the d channel is the following. Note from Eq. (44) and Fig. 3 that the commanded glide path angle γ_r acts as a constant disturbance of magnitude 2.5 deg into the system. The disturbance affects $\dot{d}$. To reject this constant disturbance, a type I system with respect to d is needed, which requires the integrator in the d feedforward path.

C. PI and Controls Design

Since the loop gain around the velocity loop is not of type I, weighting of the steady-state error will be required to force v_T to go to zero at steady state. Thus, we propose the PI

$$J = \frac{1}{2}\int_0^\infty (qt^2\tilde{e}^T\tilde{e} + \bar{u}^T\bar{u})\, dt + \frac{1}{2}v\bar{e}^T\bar{e} \qquad (64)$$
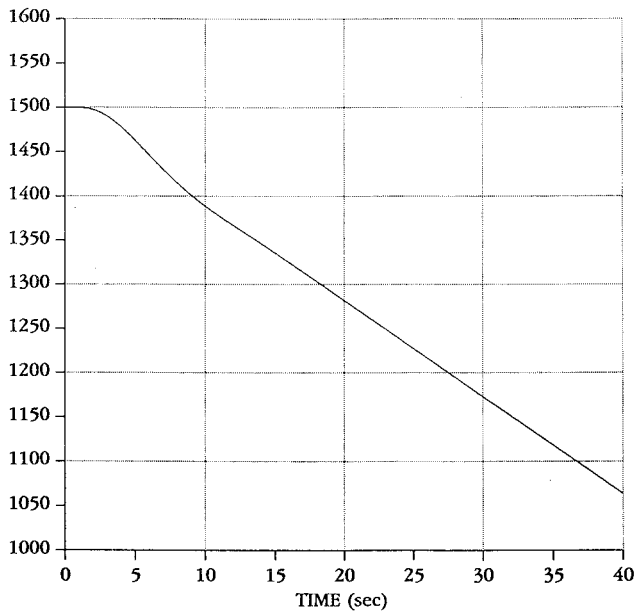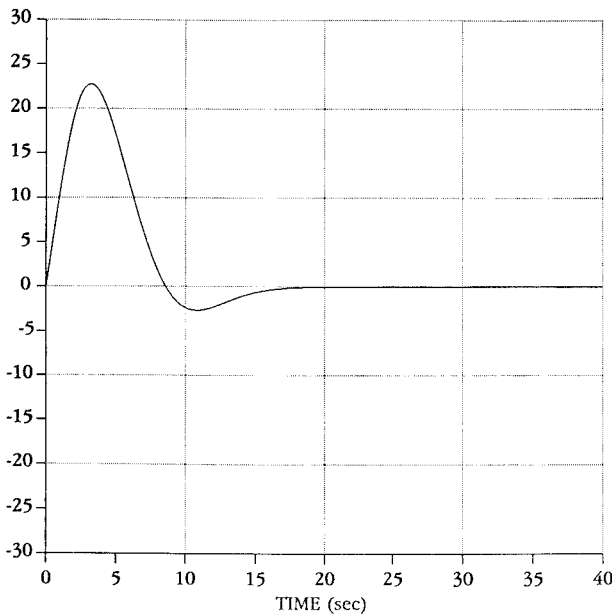
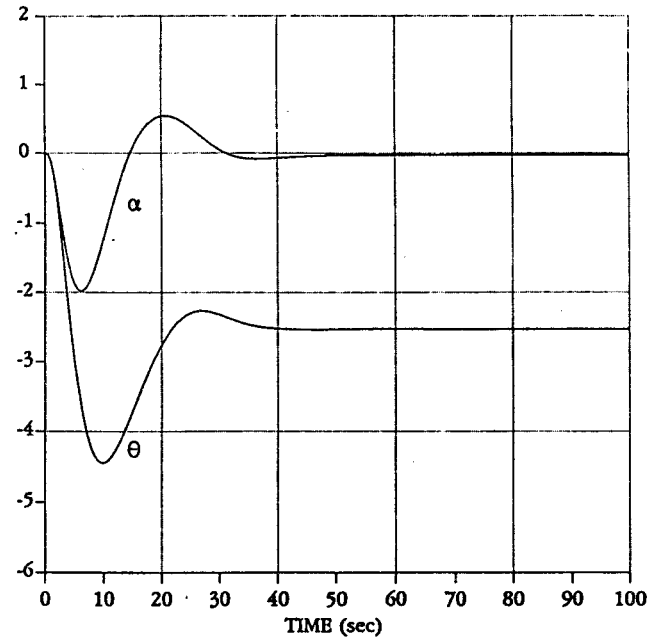Fig. 4 Altitude h (ft).

Fig. 5 Off-glide-path distance d (ft).

Fig. 6 Angle-of-attack α and pitch angle θ (deg).

The motivation for the weighting t^2 follows. Weighting $\bar{e}$ in the PI makes sense since we want it to vanish. However, $\bar{e}^T\bar{e} = \tilde{x}^TH^TH\tilde{x}$ and (H,A) is not observable. In fact, the compensator states are not observable through $z = Hx$. An LQ design without the weighting t^2 would, therefore, fail. To correct the situation, it is possible to weight the entire state in the PI by using a term such as $\tilde{x}^TQ\tilde{x}$. However, this would give too many design parameters (i.e., the elements of Q) and lead to a counterintuitive situation. Therefore, t^2 weighting is used to correct the observability problem.

After several design iterations, it was found that suitable values for the PI design parameters were $q = 0.001$ and $v = 100$. The Davidon-Fletcher-Powell algorithm[19] was used to solve for the optimal gain K using the design equations of Sec. IV. The option was selected of fixing seven of the gain elements to zero as required by Eq. (62).

With $q = 0.001$ and $v = 100$, the optimal control gains were

$$K = \begin{bmatrix} 2.598 & 0 & 0 & 0 & 0 & -0.9927 & 0 \\ 0 & 583.7 & -58.33 & -2.054 & -1.375 & 0 & 6.1 \end{bmatrix}$$

$$(65)$$

and the closed-loop poles were at

$$\begin{array}{c} -0.27 \pm j\,1.01 \\ -0.36 \pm j\,0.49 \\ -0.37 \pm j\,0.09 \\ -1.18, \quad -4.78, \quad -8.38, \quad -10.08 \end{array} \qquad (66)$$

Thus, the slowest time constant is $1/0.27 \approx 4$ s.

D. Simulation and Discussion

A simulation of the glide slope coupler appears in Figs. 4-8. The aircraft was initialized in level flight at 1500 ft. The glide slope coupler was switched on as the aircraft crossed through the glide path.

For simulation purposes, we used the exogenous input $\gamma_r = 2.5$ deg, the desired glide path angle, and reference commands of $r_v = 0$ and $r_d = 0$. Altitude h was added as a state using the equation for vertical velocity

$$\dot{h} = V_T \sin\gamma \approx V_T(\theta - \alpha) \qquad (67)$$

with V_T assumed to be the trim velocity of 250 ft/s.
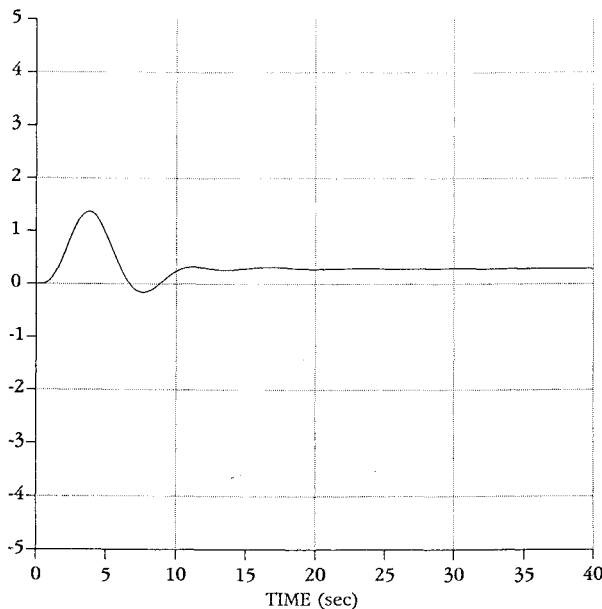
Fig. 7   Velocity deviation v_T (ft/s).
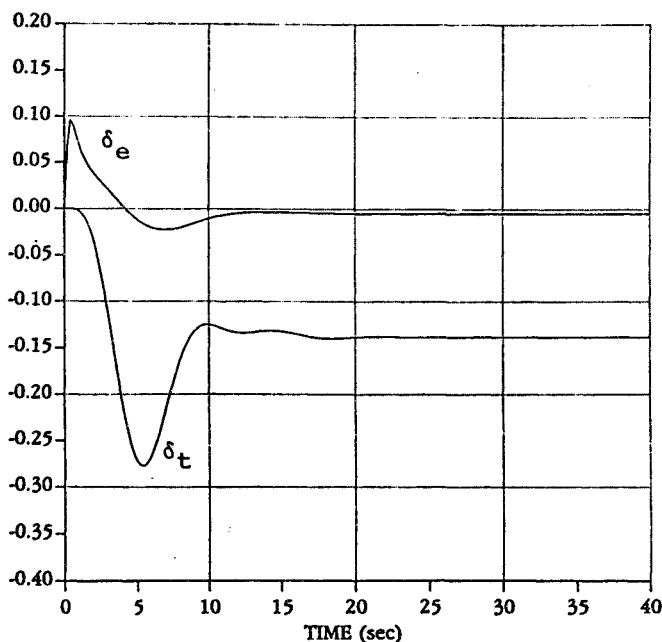
Fig. 8   Control efforts δ_e (rad) and δ_t (per unit).

According to the altitude plot in Fig. 4, after a small transient lasting about 20 s, the aircraft settles down onto the glide path and follows it down. Touchdown occurred at 137.5 s. Figure 5 shows the off-glide path error d.

The angle-of-attack α and pitch angle θ are shown in Fig. 6. Note that, at steady-state, $\gamma = \theta - \alpha = -2.68 - (-0.18) = -2.5$ deg as required on the glide path. Since, in the descending configuration the aircraft is no longer in the original trim condition, a small angle of attack α of -0.18 deg remains at steady state.

It is important to note that good performance was obtained *without adding an integrator in the velocity channel* by using the steady-state error weighting in the PI, Eq. (64). Figure 7 shows the velocity deviation v_T, whose steady-state value of 0.2 ft/s is quite small in spite of the fact that the velocity channel is of type 0. The steady-state total velocity of 250.2 ft/s is quite acceptable.

The elevator and throttle control efforts δe and δt are shown in Fig. 8. Note the coordinated control achieved in this two-in-

put system using the LQ approach. Since the descent down the glide path does not represent the original trim condition, the steady-state values of the control efforts are not zero. Intuitively, less throttle is required to maintain 250 ft/s if the aircraft is descending. Figure 8 shows that, as the aircraft passes through the glide path, the elevator control is pushed forward and the throttle is cut. As a result, the angle of attack and pitch angle decrease. After a slight positive position error d and an initial increase in velocity v_T, further control effort stabilizes the aircraft on the glide path.

With the optimal gains in Eq. (65), according to Eq. (47) the velocity channel compensator is

$$\frac{w_v}{e_v} = -0.9927 \frac{s + 2.38}{s + 5} \qquad (68)$$

which is a lead compensator as anticipated. It is important to note that the problem formulation has resulted in the *compensator zeros being selected in an optimal fashion.*

VI. Conclusions

A technique was provided for designing servo compensators of desired structure that relies on output-feedback techniques. The approach involves selecting the control gains so that *both the closed-loop poles and zeros are modified* to obtain suitable step responses. The correct initial conditions for this problem are not uniformly distributed, but have a specific form that was derived. To shape the step response for arbitrary systems, not only those of type I, a PI was proposed that weights the steady-state error. To avoid the usual problems involving observability properties in selecting PI weights, time weighting of the form t^k was also used in the PI. Necessary conditions were derived for determining the feedback gains so that an efficient gradient-based algorithm such as Davidon-Fletcher-Powell can be used to find the optimal gains. To illustrate the proposed approach, a glide slope coupler was designed.

Acknowledgment

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